Math 10A with Professor Stankova
Worksheet, Discussion #17; Wednesday, 10/4/2017
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Anti-Derivative Word Problems

Example

1. I throw a ball up into the air with an initial velocity of $10m/s$. Assuming that gravity produces a constant acceleration of $-10m/s^2$, how long will it take for the ball to come back to the ground?

> **Solution:** The initial acceleration is $-10m/s^2 = a = \frac{dv}{dt}$. Finding the general form of the antiderivative gives $v(t) = -10t + C$. We are told that initially $v(0) = -10 \cdot 0 + C = C = 10$ so $v(t) = -10t + 10 = \frac{dx}{dt}$. Finding another antiderivative gives $x(t) = -5t^2 + 10t + C$. Initially $x(t) = 0$ so $C = 0$ and hence $x(t) = -5t^2 + 10t$. Now we want to find when the ball to hit the ground, or when $x(t) = 0$. This gives us $-5t^2 + 10t = -5t(t-2) = 0$ so $t = 0$ or $t = 2$. Thus, the answer is $t = 2$ seconds.

Problems

2. An airplane starts accelerating at a rate of $4m/s^2$. After 20 seconds, it finally lifts off the ground. How far did it travel before takeoff?

> **Solution:** We know that the acceleration is $4 = a = \frac{dv}{dt}$. Taking the indefinite integral, we get that $v = 4t + C$. Initially at time $t = 0$, the plane is at rest so $v(0) = 0$ and so $4 \cdot 0 + C = C = 0$. So, $v(t) = 4t$. We know that $v(t) = \frac{dx}{dt}$ so $x = 2t^2 + C$ for some constant C. Initially, the plane has traveled a distance of 0 so $x(0) = 2 \cdot 0^2 + C = C = 0$. Therefore, we have that $x(t) = 2t^2$. After 20 seconds, we have that the plane has traveled $x(20) = 2 \cdot 20^2 = 800m$.

3. A ball is dropped from a height of $500m$. Assuming gravity is $-10m/s^2$, how long does it take for the ball to hit the ground?

Solution: We have that $a = \frac{dv}{dt} = -10$ so $v = -10t + C$. Initially it is at rest so $C = 0$. Then $v = \frac{dx}{dt} = -10t$ so $x = -5t^2 + C$. Initially, it is at a height of $500m$ so $C = 500$. We want to solve $x(t) = 0$ so $-5t^2 + 500 = -5(t^2 - 100) = 0$ so $t = 10$ seconds.

4. A biker is initially traveling $45m/s$ and starts braking with a constant deceleration of $9m/s^2$. How far does he go before he comes to a complete stop?

Solution: We are given that $a = \frac{dv}{dt} = -9$ so $v(t) = -9t + C$. Initially, we are told $v(0) = C = 45$ so $v(t) = \frac{dx}{dt} = -9t + 45$ so $x(t) = -4.5t^2 + 45t$. We want to know how far the biker travels by the time $v(t) = 0$ so when $9t = 45$ or $t = 5$ seconds. So the biker travels $x(5) = -4.5 \cdot 5^2 + 45 \cdot 5 = 112.5m$.

5. In t months from now, the population of Berkeley will be changing at a rate of $25+10t^{2/3}$. If the current population is 2000, what is the population 8 months from now?

Solution: Let $p(t)$ be the population in t months. We are given that $\frac{dp}{dt} = 25+10t^{2/3}$ so $p(t) = 25t + 6t^{5/3} + C$. We are told that $p(0) = 2000$ so $p(t) = 6t^{5/3} + 25t + 2000$. So in 8 months, the population is $p(8) = 6 \cdot 32 + 25 \cdot 8 + 2000 = 2392$.

6. In t seconds, a bacteria population will be increasing at a rate of $50e^{5t}$. If the initial bacteria population is 200, what will it be in 10 seconds?

Solution: We are told that $\frac{dP}{dt} = 50e^{5t}$ so $P(t) = 10e^{5t} + C$. We are told $P(0) = 10 + C = 200$ so $C = 190$. Therefore $P(t) = 10e^{5t} + 190$. So in 10 seconds, the population will be $10e^{50} + 190$.

7. An atom is losing energy at a rate of $10J/s$. If the atom initially has $100J$ worth of energy, how much energy will it have after 5 seconds?

Solution: Let E be the energy so $\frac{dE}{dt} = -10$ and so $E(t) = -10t + C$. We have $E(0) = C = 100$ so $E(t) = -10t + 100$. Thus after 5 seconds, we have $E(5) = 100 - 50 = 50$.

Riemann Sums

Example

8. Using limits, find the integral of x^2 from 0 to 3.

Solution: Splitting up $[0, 3]$ into n intervals gives the intervals of length $\frac{3-0}{n} = \frac{3}{n}$. So our intervals are $[0, \frac{3}{n}], [\frac{3}{n}, \frac{6}{n}], \ldots, [\frac{3n-3}{n}, \frac{3n}{n}]$. Using the right endpoints, we have that the Riemann sum is

$$R_n = \left(\frac{3}{n}\right)^2 \frac{3}{n} + \left(\frac{6}{n}\right)^2 \frac{3}{n} + \cdots + \left(\frac{3n}{n}\right)^2 \frac{3}{n}$$

$$= \frac{3 \cdot 3^2}{n^3} + \frac{3 \cdot 6^2}{n^3} + \cdots + \frac{3 \cdot (3n)^2}{n^3}$$

$$= \frac{3^3}{n^3}[1^2 + 2^2 + \cdots + n^2] = \frac{3^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{27}{6}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$$

Taking the limit, we have that

$$\lim_{n \to \infty} R_n = \frac{27}{6}[2 + 0 + 0] = \frac{54}{6} = 9.$$